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Simplifying Radicals

Unit 10 Lesson 2

Simplifying Radicals

Students will be able to:

- Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number
- Simplify expressions involving radicals using the properties of radicals.

Simplifying Radicals

Key Vocabulary:

- Radical
- Radicand
- Simplifying Radicals
 - Denominator
 - Like Radicals



Simplifying Radicals

Radicals (or roots) are the opposite operation of applying exponents.

A power can be undone with a radical and a radical can be undone with a power.

- $2^2 = 4$ so $\sqrt{4} = 2$

$$\text{index} \sqrt{\text{radicand}}$$

Radical sign

- $2^3 = 8$ so $\sqrt[3]{8} = 2$

Simplifying Radicals

Common Radicals:

- A square (second) root is written as $\sqrt{}$
- A cube (third) root is written as $\sqrt[3]{}$
- A fourth root is written as $\sqrt[4]{}$

Simplifying Radicals

- To simplify a radical, factor the expression under the radical sign to its prime factors.
- For every pair of like factors, bring out one of the factors.
- Multiply whatever is outside the sign, and then multiply whatever is inside the sign.
- Remember that for each pair, you “bring out” only one of the numbers.



Simplifying Radicals

- Variables in a radicand are simplified in the same way. You've got a pair of can be taken "out front".
- Negative Radicals

The only restriction that exists for negative signs and radicals is that there cannot be a negative sign under an even root since there is no real solution to this problem.

However, a negative sign can exist in front of a radical or under odd roots and still be able to obtain a real number.

Simplifying Radicals

- **PRODUCT RULE FOR RADICAL**

The radical of the product is the product of two radicals.

$$\sqrt[m]{xy} = \sqrt[m]{x} * \sqrt[m]{y}$$



Simplifying Radicals

QUOTIENT RULE FOR RADICAL

The radical of a quotient is the quotient of the radicals.

$$\sqrt[m]{\frac{x}{y}} = \frac{\sqrt[m]{x}}{\sqrt[m]{y}} \quad y \neq 0$$

Simplifying Radicals

Sample Problem 1: Simplify the following expressions.
Assume that all variables represent positive real numbers.

a. $\sqrt{20} =$

b. $\sqrt{72a^2} =$

c. $\sqrt{45} =$

Simplifying Radicals

Sample Problem 1: Simplify the following expressions.
Assume that all variables represent positive real numbers.

$$a. \sqrt{20} = \sqrt{4 * 5} = \sqrt{2^2} * \sqrt{5} = 2\sqrt{5}$$

$$b. \sqrt{72a^2} = \sqrt{2 * 36 * a^2} = \sqrt{2} * \sqrt{6^2} * \sqrt{a^2} = 6a\sqrt{2}$$

$$c. \sqrt{45} = \sqrt{3^2 * 5} = \sqrt{3^2} * \sqrt{5} = 3\sqrt{5}$$

Simplifying Radicals

Sample Problem 1: Simplify the following radicals. Assume that all variables represent positive real numbers.

d. $\sqrt{\frac{24}{54}} =$

e. $\sqrt[5]{\frac{-32y^5}{x^{10}}} =$

f. $\sqrt[3]{\frac{xz^4}{y^6}} =$



Simplifying Radicals

Sample Problem 1:

Simplify the following radicals. Assume that all variables represent positive real numbers.

$$\text{d. } \sqrt{\frac{24}{54}} = \frac{\sqrt{24}}{\sqrt{54}} = \frac{\sqrt{4*6}}{\sqrt{9*6}} = \frac{\sqrt{2^2}}{\sqrt{3^2}} = \pm \frac{2}{3}$$

$$\text{e. } \sqrt[5]{\frac{-32y^5}{x^{10}}} = \frac{\sqrt[5]{(-2)^5*y^5}}{\sqrt[5]{x^5}*\sqrt[5]{x^5}} = \frac{-2y}{x^2}$$

$$\text{f. } \sqrt[3]{\frac{xz^4}{y^6}} = \frac{\sqrt[3]{xz^4}}{\sqrt[3]{y^6}} = \frac{\sqrt[3]{x}*\sqrt[3]{z}*\sqrt[3]{z^3}}{\sqrt[3]{y^3}*\sqrt[3]{y^3}} = \frac{z\sqrt[3]{xz}}{y^2}$$

Simplifying Radicals

- Radicals can also be written in exponent notation. However, in this case the exponent would be a fraction.

$$\sqrt[n]{x^m} = x^{\frac{m}{n}}$$

- If n is odd then $\sqrt[n]{x^n} = x$
- If n is even then $\sqrt[n]{x^n} = |x|$

Simplifying Radicals

Sample Problem 2:

Write each expression in radical notation.

a. $(4)^{\frac{1}{2}} =$

b. $(-8)^{\frac{1}{3}} =$

c. $4^{\frac{1}{3}} =$

Simplifying Radicals

Sample Problem 2:

Write each expression in radical notation.

a. $(4)^{\frac{1}{2}} = \sqrt{4} = \pm 2$

b. $(-8)^{\frac{1}{3}} = \sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2$

c. $4^{\frac{1}{3}} = \sqrt[3]{4}$

Simplifying Radicals

Sample Problem 3:

Write each expression in exponential notation.

a. $\sqrt[3]{-64} =$

b. $\sqrt[5]{32} =$

c. $\sqrt[4]{625} =$

Simplifying Radicals

Sample Problem 2:

Write each expression in exponential notation.

$$\text{a. } \sqrt[3]{(-64)} = (-64)^{\frac{1}{3}} = (-4)^{3 \cdot \frac{1}{3}} = -4$$

$$\text{b. } \sqrt[5]{32} = 2^{5 \cdot \frac{1}{5}} = 2$$

$$\text{c. } \sqrt[4]{625} = \sqrt[4]{5^4} = \pm 5^{\frac{4}{4}} = \pm 5$$

Simplifying Radicals

RATIONALIZING THE DENOMINATOR

When dealing with fractions, a final answer cannot contain radicals in the denominator.

Therefore, it is necessary to eliminate any radical from the denominator.

The process of removing the radical from the denominator is called **rationalizing the denominator**.

Simplifying Radicals

Rationalizing the denominator

1. An expression is considered simplified only if there is no radical sign in the denominator.

$$\frac{1}{\sqrt[n]{x^m}} = \frac{1}{\sqrt[n]{x^m}} * \frac{\sqrt[n]{x^{n-m}}}{\sqrt[n]{x^{n-m}}} = \frac{\sqrt[n]{x^{n-m}}}{x}$$

2. Rationalizing the denominator with two terms, one or both of which involve square root.

Step 1: Multiply the numerator and denominator by the conjugate of the denominator

Step 2: Simplify the resulting expression if possible

Simplifying Radicals

Sample Problem 4:

Simplify the following expressions. Assume that all variables represent positive real numbers.

a. $\frac{7}{\sqrt{5}} =$

b. $\frac{2}{\sqrt{(x+y)}} =$

Simplifying Radicals

Sample Problem 4:

Simplify the following expressions. Assume that all variables represent positive real numbers.

$$\text{a. } \frac{7}{\sqrt{5}} = \frac{7}{\sqrt{5}} * \frac{\sqrt{5}}{\sqrt{5}} = \frac{7\sqrt{5}}{5}$$

$$\text{b. } \frac{2}{\sqrt{(x+y)}} = \frac{2}{\sqrt{(x+y)}} * \frac{\sqrt{(x+y)}}{\sqrt{(x+y)}} = \frac{2\sqrt{(x+y)}}{(x+y)}$$

Simplifying Radicals

Sample Problem 4:

Simplify the following expressions. Assume that all variables represent positive real numbers.

c. $\frac{2}{2-\sqrt{6}} =$

d. $\frac{x}{1-\sqrt{x}} =$

Simplifying Radicals

Sample Problem 4:

Simplify the following expressions. Assume that all variables represent positive real numbers.

$$\text{c. } \frac{2}{2-\sqrt{6}} = \frac{2}{2-\sqrt{6}} * \frac{2+\sqrt{6}}{2+\sqrt{6}} = \frac{2(2+\sqrt{6})}{2^2-(\sqrt{6})^2} = \frac{2(2+\sqrt{6})}{-2} = -(2 + \sqrt{6})$$

$$\text{d. } \frac{x}{1-\sqrt{x}} = \frac{x}{1-\sqrt{x}} * \frac{1+\sqrt{x}}{1+\sqrt{x}} = \frac{x(1+\sqrt{x})}{1^2-(\sqrt{x})^2} = \frac{x(1+\sqrt{x})}{1-x}$$

Simplifying Radicals

- **POWER OF A RADICAL**

The radicand has to be raised to the exponent of the power while the index remains the same.

$$\left({}^m\sqrt{x^n}\right)^p = {}^m\sqrt{x^{n*p}}$$

Simplifying Radicals

- **RADICAL OF A RADICAL**

The radicand stays the same, and the index is the product of the indices.

$${}^m\sqrt{{}^n\sqrt{x}} = {}^{m*n}\sqrt{x}$$

Simplifying Radicals

Sample Problem 5: Simplify the following expressions. Assume that all variables represent positive real numbers.

a. $(\sqrt[5]{125})^2 =$

b. $\sqrt[3]{\sqrt{x^5}} =$

Simplifying Radicals

Sample Problem 5: Simplify the following expressions.
Assume that all variables represent positive real numbers.

$$\text{a. } \left(\sqrt[5]{125} \right)^2 = \sqrt[5]{5^{3 \cdot 2}} = \sqrt[5]{5^6} = 5 \sqrt[5]{5}$$

$$\text{b. } \sqrt[3]{\sqrt{x^5}} = \sqrt[3 \cdot 2]{x^5} = \sqrt[6]{x^5}$$

Simplifying Radicals

Like radicals are radicals that have the same index and the same radicand

Simplifying Radicals

Sample Problem 6:

Simplify radicals and recognize like or unlike radicals.

a. $2\sqrt{5}$ and $3\sqrt{5}$

b. $\sqrt{125}$; $3\sqrt{80}$ and $\sqrt{45}$

Simplifying Radicals

Sample Problem 6:

Simplify radicals and recognize like or unlike radicals.

a. $2\sqrt{5}$ and $3\sqrt{5}$

LIKE RADICALS

b. $\sqrt{125}$; $3\sqrt{80}$ and $\sqrt{45}$
 $\sqrt{25 * 5}$; $3\sqrt{16 * 5}$; and $\sqrt{9 * 5}$
 $5\sqrt{5}$; $12\sqrt{5}$; and $3\sqrt{5}$

LIKE RADICALS

Simplifying Radicals

Sample Problem 6:

Simplify radicals and recognize like or unlike radicals.

c. $4\sqrt[3]{x^2}$ and $4\sqrt{x^3}$

d. $3\sqrt{ab^2}$; $4b\sqrt{a}$; $\sqrt{4ab^2}$

Simplifying Radicals

Sample Problem 6:

Simplify radicals and recognize like or unlike radicals.

c. $4\sqrt[3]{x^2}$ and $4\sqrt{x^3}$

UNLIKE RADICALS

d. $3\sqrt{ab^2}$; $4b\sqrt{a}$; $\sqrt{4ab^2}$
 $3b\sqrt{a}$; $4b\sqrt{a}$; $2b\sqrt{a}$

LIKE RADICALS